Their trustworthiness when applied to individual families is shown as strongly in Table IV, whose results are conveniently summarised in Table VI. I have there classified the amounts of error in the several calculations: thus if the estimate in any one family was 3 light-eyed children and the observed number was 4, I should count the error as 1·0. I have worked to one place of decimals in this table, in order to bring out the different shades of trustworthiness in the three sets of calculations, which thus become very apparent. It will be seen that the calculations in Class III are by far the most precise. In more than one-half of those calculations the error does not exceed 0·5, whereas in I and II more than three-quarters of them are wrong to at least that amount. Only one-quarter of Class III are more than 1·1 in error, but somewhere about the half of Classes I and II are wrong to that amount. In comparing I with II, we find I to be slightly, but I think distinctly, the superior estimate. The relative accuracy of III as compared with I and II, is what we should have expected, supposing the basis of the calculations to be true, because the additional knowledge utilised in III, over what is turned to account in I and II, must be an advantage.

Conclusion.—The general trustworthiness of these calculations of the probable proportion of light-eyed and dark-eyed children in individual families, whose ancestral eye-colour is more or less known, is comparable with the chance of drawing a white or a black ball out of a bag in which the relative numbers of white and black balls are the same as those given by the calculation. The larger the proportion of data derived from a certain knowledge of ancestral eye-colours, and not from inferences about them, the more true does the comparison become. My returns are insufficiently numerous and too subject to uncertainty of observation to make it worth while to submit them to a more rigorous analysis, but the broad conclusion to which the present results irresistibly lead, is that the same peculiar hereditary relation that was shown to subsist between a man and each of his ancestors in respect to the quality of stature, also subsists in respect to that of eye-colour.

II. "A General Theorem in Electrostatic Induction, with Application of it to the Origin of Electrification by Friction." By John Buchanan, B.Sc., Demonstrator of Physics, University College, London. Communicated by Professor G. Carey Foster, B.A., F.R.S. Received May 13, 1886.

Part I.

This paper contains the results of an investigation into the question: If a dielectric be brought into a field of electric force, and there its

specific inductive capacity is changed, what will be the electrical condition of the dielectric? The subject has occupied me both in its theoretical and experimental aspects for a considerable time, and I believe that the answer to the question throws light upon some fundamental electrical phenomena.

This investigation has led me to a general theorem in electrostatic induction which may be stated as follows:—

When a dielectric is brought into a field of electric force and the specific inductive capacity is there altered, in general the dielectric becomes electrified.

To give definiteness to our notions, let us imagine a field of electric force to be due to an electrified conductor, which we will call the "primary;" inclosing this primary is a conducting shell which is connected to earth.

For simplicity we will assume, for the present, that the charge on the primary remains unchanged in magnitude during this series of operations:—

(1.) The dielectric is brought into the field of force;
(2.) The specific inductive capacity is *increased;*
(3.) The dielectric is carried out of the field.

The state of the field is exactly the same as it was before the operations were performed. We can therefore fix our attention on the dielectric.

Let us compare the work done *by* electrical forces with the work done *against* them in the operations (1), (2), (3). We have in (1) work done *by* electrical forces in assisting to bring the dielectric into the field; work is also done *by* (say) the forces in (2). In operation (3) work is done *against* electrical forces. The question to be answered is this, does the following equation hold in every case?

Work done by electrical forces in bringing the dielectric into the field
+ work done by the forces during the change of specific inductive capacity
= work done against the electrical forces in carrying the dielectric out of the field.

If this equation be true under all circumstances, there is no excess of work done by or against electrical forces. We would have then no reason to expect to find an electric distribution on or in the dielectric, whose energy would be the equivalent of the excess of work done. Now that the above equation should always hold seems to me at variance with sound conceptions regarding the effect of an arbitrary change in the physical state of a body.

Take, for instance, a case such as that of a piece of hot glass left to

cool, and meanwhile to undergo electrolysis by the action of the electric forces of the field; and when cold carried out of the field.

The important part here played by the element *time*, renders it quite impossible to maintain *à priori* that the above hypothetical equation should hold under all circumstances: the proof would need to be experimental.

The investigation given below is designed to express in definite terms the effect of the somewhat general conditions therein specified.

Let us denote the potential of the primary by V, its charge by q; the specific inductive capacity of the dielectric placed in the field of force by K; and the electrostatic capacity of the whole system by C. Then the theorem is that the magnitude and sign of the "apparent electrification" of the dielectric are given by an equation of the form—

$$h = -\left(\frac{d\pi}{dV} + V\frac{dC}{dK}\right),$$

where h denotes the rate of change of the apparent electrification of the dielectric with regard to the specific inductive capacity K as independent variable; and π denotes the rate of change of the work done against electrical forces with regard to the same independent variable.

By translating the theorem into the language of magnetism, we obtain a theorem relating to magnetic induction in matter placed in a magnetic field of force.

Proof.

The dielectric being supposed in the field of force, let the specific inductive capacity be changed. The influence of this change of specific inductive capacity of the dielectric on the electrical state of the primary can be expressed by taking as independent variables the potential V of the primary, and the specific inductive capacity K of the dielectric. Due to an arbitrary change of potential δV, and an arbitrary change of specific inductive capacity δK, there will be an augmentation δq of the charge of the primary—by connecting it to proper sources of electricity—given by an equation of the form—

$$\delta q = C\,.\,\delta V + V\,.\,\frac{dC}{dK}\,.\,\delta K + h\,.\,\delta K\,. \quad . \quad . \quad . \quad . \quad (1.)$$

The first term of the right hand member expresses the well-known relation between the charge, the potential, and the capacity of an electrical system; the second term expresses the effect of the change of capacity caused by the alteration of specific inductive capacity; and the third term expresses the effect of the electrification of the dielectric due to the same cause. What I propose to show is, that

the quantity h need not be zero, unless under very special circumstances.

As it appears in (1), $h\,.\,\delta\mathrm{K}$ is clearly the quantity of electricity that must be given to the primary in order to maintain the potential constant whilst the specific inductive capacity is altered by $\delta\mathrm{K}$, and this in addition to the influence of the mere change of capacity of the system.

We may assume as a well-known result that for a closed cycle of operations

$$\int \delta q = 0,$$

and δq is a perfect differential.*

Expressed in words, this is equivalent to stating that, when after undergoing a series of changes, the potential is brought back to any given value V, and the molecular condition of the dielectric in the field of force is brought back to its initial state, then the charge of the primary is the same as at first.

The analytical statement of the condition that δq in (1) is a perfect differential gives us—

$$\frac{d\mathrm{C}}{d\mathrm{K}} = \frac{d}{d\mathrm{V}}\left(\mathrm{V}\frac{d\mathrm{C}}{d\mathrm{K}} + h\right),$$

or

$$\frac{dh}{d\mathrm{V}} + \mathrm{V}\,.\,\frac{d}{d\mathrm{V}}\left(\frac{d\mathrm{C}}{d\mathrm{K}}\right) = 0. \qquad (2.)$$

In order to obtain another relation between the quantities, let us denote by $\delta\epsilon$ the increment of electrical energy of the system during the series of operations described above as leading to (1). This is expressed by an equation of the form—

$$\delta\epsilon = \mathrm{V}\delta q + \pi\delta\mathrm{K}. \qquad (3)$$

The meaning of the first term of the right hand number of (3) is obvious; the other term, $\pi\delta\mathrm{K}$, denotes the work done *against* electrical forces when the specific inductive capacity of the dielectric is *increased* by $\delta\mathrm{K}$.

By the principle of the conservation of energy, for a closed cycle of operations

$$\int \delta\epsilon = 0,$$

and $\delta\epsilon$ is a perfect differential. Hence, if we express the analytical condition of this, after putting for δq its value from (1), we get—

* I would here acknowledge my great indebtedness to a paper on the "Conservation of Electricity," by M. G. Lippmann, "Annales de Chimie et de Physique," 5[me] Sér., T. 24 (1881) p. 145.

$$\frac{d}{dV}\left(V^2 . \frac{dC}{dK} + hV + \pi\right) = \frac{d}{dK}(VC).$$

Performing the differentiations and making use of (2) we have—

$$\frac{d\pi}{dV} + h + V\frac{dC}{dK} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

If (4) be differentiated with respect to V and (2) be again applied, we find—

$$\frac{d^2\pi}{dV^2} + \frac{dC}{dK} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

Hence, finally, the theorem can be expressed in either of the forms—

$$h = -\left(\frac{d\pi}{dV} + V . \frac{dC}{dK}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

$$h = -\left(\frac{d\pi}{dV} - V . \frac{d^2\pi}{dV^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (6'.)$$

Since as a rule π will probably increase with V, $\frac{d\pi}{dV}$ will usually have the same sign as π.

The form (6′), amongst other uses, enables us to get at once an important result, viz., the circumstances under which h is zero. We have $h = 0$ when—

$$\frac{d\pi}{dV} - V . \frac{d^2\pi}{dV^2} = 0.$$

Integrating twice we get successively—

$$\frac{d\pi}{dV} = aV,$$

where a is an arbitrary constant; and

$$\pi = \tfrac{1}{2}aV^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

The constant of the second integration will in general be zero.

Equation (7), taken in connexion with (5), gives by differentiation,

$$\frac{dC}{dK} = \text{const.} = -a.$$

It appears therefore that in order to have no electrification of the dielectric when the specific inductive capacity is altered, the change of capacity of the system must be proportional to the change of specific inductive capacity.

Remark also that since $\frac{dC}{dK}$ must in general be positive, the quantity a, and therefore also π, must be negative, by (7).

It is not difficult to prove that the condition $\frac{dC}{dK}$ = const. leads to the conclusion that the whole electric field must be occupied by an electrically homogeneous dielectric.

The following proof seems to be convenient. Let us imagine the assumed heterogeneous medium to consist of shells of dielectric material whose boundaries are equipotential surfaces; each shell is supposed to be itself homogeneous. If the bounding equipotential surfaces consisted of excessively thin conducting shells, the distribution of electric force in the field would be unaltered. Each consecutive pair of conducting equipotential surfaces with the (homogeneous) shell of dielectric between, would then form a condenser. And since the same quantity of electric induction crosses all the equipotential surfaces in the field, the capacity of the whole system would be simply that of a series of condensers in "cascade."

When air is the dielectric, denote the capacity of the condenser which consists of the primary, the first conducting equipotential surface, and the space between, by C_1; the capacity of the condenser formed by the first and second equipotentials by C_2, and so on. If, instead of air, the spaces be respectively filled up by dielectrics of specific inductive capacities K_1, K_2, and so on, we have for the capacity of the whole system C, the relation—

$$\frac{1}{C}=\frac{1}{K_1C_1}+\frac{1}{K_2C_2}+\ \ldots$$

Replacing the shells of dielectric by others of different specific inductive capacity, and denoting the changed quantities by dashes, we have—

$$\frac{1}{C'}=\frac{1}{K_1'C_1}+\frac{1}{K_2'C_2}+\ \ldots$$

Let now $\qquad K_1'-K_1=K_2'-K_2=\ \ldots\ =\delta K,$

so that the alteration of specific inductive capacity is the same for all the dielectrics.

It is evident that unless—

$$K_1=K_2=K_3=\ \ldots$$

and $\qquad K_1'=K_2'=\ \ldots$

it is impossible that—

$$\left(\frac{C'-C}{\delta K}\right)$$

should be independent of δK.

The conclusion given above follows at once.

To sum up the discussion, the result is that the equations (5), (6), and (6′) express the effect of heterogeneity in the constitution of the dielectric medium.

Note.—It may be well to notice here an objection that might be raised against the validity of the above theorem. It could be urged, that since Dr. Hopkinson has found by experiment* that no change of specific inductive capacity could be detected when glass was subjected to electric stress varied through a very wide range of magnitudes, the quantity π in the theorem has no existence. The experiments just referred to, however, only prove that the quantity π is very small. It is shown in Part II of this paper that for most substances π has a value different from zero, being positive in some cases, negative in others.

Part II.

Application to the Theory of the Origin of Electrification by Friction.

The rubbing together of two bodies is the most ancient means known of obtaining electricity. The absence of any accepted explanation of this historical mode of rendering a body electrified does not need to be enlarged upon.

I have ventured to entertain the hope that the general theorem proved above, together with the experimental results obtained by Dr. Kerr in his memorable researches in the region of "electro-optics," may be found to prove adequate to the explanation of the fundamental and important subject of electrification by friction.

As is well known, Dr. Kerr has proved that transparent dielectrics become as a rule doubly refracting when subjected to electric force. Under the action of electric stress, a dielectric becomes strained. With the electric stress different at different parts of the field of force, the strain varies from point to point. This space-variation of strain manifests itself optically by the material assuming the property of converting plane polarised into elliptically polarised light, when the incident light is passed transversely across the direction of the electric induction in the dielectric, and the plane of polarisation is inclined at an angle to this direction.

Moreover, as has been pointed out and proved experimentally by Prof. Quincke,† the electrically-induced strain—the effect of which Dr. Kerr observed as double refraction—produces a change in the index of refraction. When the strain is uniform, Quincke has shown that no double refraction ensues. Doubly refracting properties are

* "Phil. Trans.," vol. 172, p. 355 (1881).

† Quincke, "On Electrical Expansion," "Phil. Mag.," Ser. 5, vol. x, p. 30.

assumed only in a field of force which is not uniform from point to point.

Dr. Kerr has made the remarkable discovery that some dielectrics become optically "positive," others "negative," when subjected to electric stress. I think it may be inferred from Prof. Quincke's experiments just referred to, that those bodies which Dr. Kerr found to be "positive" have their index of refraction *decreased* by electric stress; "negative" bodies on the contrary have their index of refraction *increased*. I am not aware that this point has been decided, but I hope shortly to investigate it in the laboratory of University College, London.

The sign of the change of index of refraction is not essential to the present discussion. We will assume, however, simply for convenience of statement, that a "positive" dielectric experiences a *decrease*, and a "negative" dielectric experiences an *increase* of index of refraction when placed in a field of electric force.

Now, whatever opinion may be held concerning the electro-magnetic theory of light, there can be no doubt that along with change of index of refraction of a dielectric, there goes always change of specific inductive capacity. With the supposition we have made above regarding the sign of the change of index of refraction produced in the dielectrics examined by Dr. Kerr, his results when expressed in electrical terms translate into the statements that: a "positive" dielectric has its specific inductive capacity *decreased* by electric force; a "negative" dielectric has its specific inductive capacity *increased* by electric force. In view of the theorem proved in Part I of this communication, this form of statement is very important.

It means that if the specific inductive capacity of a "positive" body be decreased in presence of a field of force, then the electric forces assist this change—work is done *by* these forces. On the other hand, if the specific inductive capacity be increased, work is done against the forces of the field.

We get corresponding statements for "negative" bodies by changing signs.

Let us return now to equation (6). It is—

$$h = -\left(\frac{d\pi}{dV} + V \cdot \frac{dC}{dK}\right).$$

Let us suppose that the dielectric is placed in a field of zero force. Then, with the disposition of apparatus that we assumed at the beginning of Part I, $V=0$, and the second term of the right hand number is zero. But the first term $\frac{d\pi}{dV}$ need not necessarily vanish

with V. Let us denote by $\left(\frac{d\pi}{dV}\right)_0$ the value of $\frac{d\pi}{dV}$ when V=0; then according to the view adopted in this paper, $\left(\frac{d\pi}{dV}\right)_0$ is a quantity which has a value characteristic of each material, and may be regarded as a property of each material in the same sense as, for instance, the index of refraction.

Let now the specific inductive capacity of the dielectric be increased by δK. Thus the *tendency* is for the dielectric to become electrified with a quantity of electricity—

$$h\delta K = -\delta K\left(\frac{d\pi}{dV}\right)_0.$$

This tendency being equal in all directions there is no resultant electrification. If, however, another dielectric is put into close contact with the first, dissymmetry is introduced. Denoting by the suffixes ($_1$) and ($_2$) the quantities relating to the two dielectrics, and by ΔE the electrification, we have initially,

$$\left.\begin{aligned}\Delta E_1 &= -\delta K_1\left(\frac{d\pi_1}{dV}\right)_0 + \delta K_2\left(\frac{d\pi_2}{dV}\right)_0 \\ \Delta E_2 &= +\delta K_1\left(\frac{d\pi_1}{dV}\right)_0 - \delta K_2\left(\frac{d\pi_2}{dV}\right)_0\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8.)$$

These two equations express my view of the mode in which electrification begins when two dielectrics are put into contact and their specific inductive capacities are altered. The change of specific inductive capacity may take place either by pressure or by friction—with liquids it is probable that only the heating effect of friction can influence the results.

According to what law the electrification goes on increasing when once started is a point still to be cleared up, the value for any material of the quantity π having still to be worked out experimentally.

Before discussing (8) it is convenient here to notice that when two bodies are in very close contact, the capacity of the system that consists of the two opposed surfaces and the extremely small distance between them, must be very great indeed. If then at any moment Q be the charge on either of these opposed surfaces and C denote the capacity of the system, then (6) becomes—

$$\begin{aligned}h &= -\left(\frac{d\pi}{dV} + \frac{Q}{C}\cdot\frac{dC}{dK}\right) \\ &= -\frac{d\pi}{dV}, \text{ nearly,}\end{aligned}$$

when $\frac{dC}{dK}$ and Q are finite and C is extremely large. Hence, in considering what is happening when two bodies are rubbed together, we need only take account of the value of $-\frac{d\pi}{dV}$ for each.

To simplify discussion of (8) we will take the second body as "neutral"; *i.e.*, $\left(\frac{d\pi}{dV}\right)_0 = 0$. We shall see that boxwood appears nearly to fulfil this condition. Also for brevity and convenience, we will put $\frac{d\pi}{dV} = \alpha$.

Two cases arise for consideration.

Case I.

$\frac{d\pi}{dV}(\alpha)$ positive, *i.e.*, work is done against electric forces by increasing the specific inductive capacity.

(*a*). "Positive" liquids. If a liquid dielectric be warmed, the index of refraction, and therefore the specific inductive capacity, is decreased. Hence, by friction there is a change of specific inductive capacity $-\delta K$ to be expected. Using ΔE in the same sense as in (8)—

$$\Delta E = +\alpha_0 \delta K.$$

It is shown by the experimental results quoted below, that ΔE positive indicates that by friction this class of liquids tends to become positively electrified. And since in this particular case the sign of ΔE is the same as that of the electrification, it ought to hold in general. It will be seen that this is true.

(*b*). "Positive" solids. Here friction, by raising the temperature of the surface, tends to change the specific inductive capacity by an amount $+\delta K$,

$$\therefore \Delta E = -\alpha_0 \delta K.$$

Such bodies tend to become negatively electrified by friction.

Case II.

$\frac{d\pi}{dV}(\alpha)$ negative. Here work is done by electrical forces when the specific inductive capacity is increased.

(*c*). "Negative" liquids. Friction tends to decrease the specific inductive capacity by an amount $-\delta K$,

$$\therefore \Delta E = -\alpha_0 \, . \, \delta K.$$

Hence "negative" liquids tend to become negatively electrified by friction.

(*d*). "Negative" solids. Friction tends to increase the specific inductive capacity,

$$\therefore \Delta E = +\alpha_0 . \delta K.$$

"Negative" solids therefore tend to become positively electrified by friction.

The conclusions under (*a*), (*b*), (*c*), (*d*) are all found to be verified by experiment.

Professor Foster has suggested to me that, in connexion with the ideas expressed by equation (8), it is interesting to find the statement by Beccaria* that the cause of the electrical difference set up between two pieces of similar silk ribbon when rubbed together lies in the unequal warming of the opposed surfaces. The oft-quoted experiments of Faraday with a feather and piece of canvas fall obviously under the same head.

I may perhaps be allowed to cite in addition some very old experiments with glass made by Bergman.† On rubbing two similar strips of glass together, the portion of the surface of either strip which received the greatest amount of friction per unit area became positive. This agrees perfectly with what may be deduced from (8). For the two strips, $\frac{d\pi}{dV} = -\alpha$ was the same; hence by (8)—

$$\therefore \Delta E_1 = \alpha_0(\delta K_1 - \delta K_2).$$

If $\delta K_1 > \delta K_2$ we get ΔE_1 positive, as Bergman found.

Experiments on the Electrification of Steam by Friction.

The experimental results given below are far from complete; but I venture to publish them as affording in some measure an experimental verification of the ideas put forward in Part I.

The method of experimenting and the arrangement of apparatus employed were essentially the same as were used by Faraday in his classical experiments on this same subject.‡

It is needless for me to say how very much I am indebted to Faraday's observations during the whole course of this experimental enquiry. Like all that the great experimenter undertook, the record of his observations in the "Researches" is a treasure-house for later experimenters to draw supplies from.

To generate steam, a small vertical copper boiler was used, which was heated by gas led to the burner by india-rubber tubing. By placing the boiler on small blocks of shellac the insulation was found at all times to be excellent. The weather was very favourable.

* Riess, "Reibungs-Electricität," § 914.

† *Ibid.*, § 913, fig. 175.

‡ "Experimental Researches," § 2075, *et seq.*

The electrical condition of the boiler was the thing tested in all the experiments: a gold leaf electroscope connected to it served as indicator.

The steam was led from the boiler through a straight brass tube about 1·2 cm. diameter and 120 cm. long, to a steam-globe of copper 10 cm. in diameter. This steam-globe was always kept well supplied with distilled water, as Faraday points out how essential it is to have the steam wet. A "feeder-tube" was used to contain the substance to be experimented upon: it was of glass 1·5 cm. internal diameter, and about 15 cm. long. A short length of narrow tubing, furnished with a glass stopcock, was fused to the main tube at the centre and at right angles to the axis. It was used to renew the supply of material in the wider tube below along which the current of steam was passed to sweep the material away. This arrangement was found convenient in working; and it possessed the great merit of allowing the pieces to be easily cleaned.

For the friction-piece that was rubbed by the current of fluid, I worked principally with a boxwood tube of as nearly as was convenient the dimensions of the tube described by Faraday as an "excellent exciter."*

By a fortunate chance, this tube was found to be very nearly at the neutral line where Dr. Kerr's "positive" and "negative" substances meet.

In order to find out if possible what was the meaning of some anomalous results that appeared in the experiments, a number of observations were made with, amongst others, tubes of pine, hawthorn, birch, sulphur, plaster-of-paris, and a tube formed out of a piece of carbon rod 1·2 cm. diameter, such as is used in electric lighting. The results obtained with the sulphur and plaster-of-paris tubes were interesting, but in no way decisive, as the tubes were found to have become very much disintegrated by the action of the current of steam; they are, therefore, not recorded in what follows. The results with the hawthorn tube will illustrate the effect of friction-tubes of materials whose place is on the negative side of the neutral line; carbon stands on the positive side.

Between the two lies the boxwood tube. This is well shown by the results obtained with methylic alcohol and amylic alcohol as shown in the list given below.

The wooden tubes used were always kept well soaked with distilled water.

After each day's work with the apparatus, it was taken to pieces, and the copper steam-globe and the feeder-tube were left to steep in a strong solution of carbonate of soda; then they were well rinsed out with distilled water before being used again. As occasion required,

* "Experimental Researches," footnote, § 2102.

methylated spirit was used to give the apparatus a thorough cleansing from all traces of oil, &c.

As a valuable test of the proper working of the apparatus, oil of turpentine was constantly in use. If everything was going well, the effect of adding a small quantity of the oil to the distilled water in the feeder-tube was to make the boiler positive, and the steam negative. On continuing to blow out, the boiler quickly passed on to negative.

This was repeated as a rule between the testing of each pair of substances.

It will be observed in the list given below that there are three persistent apparent exceptions to the rule that holds for all the other substances tried; these are turpentine, sperm oil, and chloroform.

The extremely uncertain composition of the first and second of these three bodies did not allow their exceptional behaviour to assume much importance in my eyes. But that chloroform should remain an exception to the rule appears to indicate that either the influence of the water masks that of the chloroform (*vide* remarks by Faraday on Alcohol, "Exper. Res." §2115, 2116), or that there has been a change of sign in the electro-optical position of the body due to rise of temperature. These points, together with some others that have been raised in my mind in connexion with the present application of the theorem of Part I, I hope to be able to clear up by experiments in a different direction from those recorded here. It may be desirable to state also, that I began experiments in which dry compressed air was used instead of steam, but was not able to continue them.

In conclusion I desire to record my thanks to Prof. Foster, for the valuable criticisms with which he has favoured me during the preparation of this paper.

Note.—In the experimental results which follow, the sign of the electrification is that assumed by the body to whose name it stands opposite when rubbed on the material whose name is at the top of the column.

Experimental Results (November, 1884).

Name of dielectric.	Sign of electro-optical effect.	Hawthorn tube. Sign of electrification.	Boxwood tube. Sign of electrification.		Carbon tube. Sign of electrification.
			First set of experiments.	Last set of experiments.	
Distilled water	+	+	+	+	+
Carbon disulphide..	+	..	+	..	+
Paraffin wax	+	..	+	..	..
Naphthaline	+	..	+	..	..
Lubricating oil (petroleum?) ..	?	..	+	..	..
Methyl iodide	+	..	+	..	..
Oil of turpentine. ..	+	—	—	—	—
Sperm oil	+	—	—	..	—
Benzole	+	..	+	..	+
Caoutchoucine	+	..	+	..	—
Spermaceti	+	..	+	..	..
Methylic alcohol ...	+	+	+	+	—
Amylic alcohol	—	+	—	+	—
Chloroform	—	+	+	+	+
Glycerine	—	..	—	..	..
Colza oil	—	..	—	..	..
Castor oil	—	..	—	..	..
Olive oil	—	..	—	..	..
Cod-liver oil...... ...	—	..	—	..	..

To the above may be added a list of solids whose electro-optical positions have been determined by Dr. Kerr. The electrical position of the first two is well known. The others I examined by rubbing them with the same boxwood tube as was employed in the experiments with fluids.

Name of dielectric.	Sign of the electro-optical effect.	Sign of electrification when rubbed by boxwood.
Glass	—	+
Quartz....................	—	+
Resin	+	—
Sulphur	+	—
Solid paraffin wax	+	—
Spermaceti	+	—
Naphthaline	+	—

References to Dr. Kerr's papers on electro-optics :—

"Philosophical Magazine," Ser. 4, vol. 50, 1875; pp. 337 and 446.
" " Ser. 5, vol. 8, 1879; pp. 85 and 229.
" " " 13, 1882; pp. 153 and 248.

Faraday gives a list of bodies with the sign of the electrification they acquired by friction, which is here reproduced for comparison with my results.

Name of dielectric.	Sign of the electro-optical effect.	Sign of electrification by friction.
Distilled water	+	+
Carbon disulphide	+	+
Naphthaline	+	+
Naphtha	+	+
Caoutchoucine	+	+
Spermaceti	+	—
Oil of turpentine	+	—
Resin (dissolved in alcohol)	+	—
Alcohol (ethyl)	—	—
Lard	—	—
Beeswax	—	—
Castor oil	—	—
Olive oil	—	—

III. "Notes on Alteration induced by Heat in certain Vitreous Rocks; based on the Experiments of Douglas Herman, F.I.C., F.C.S., and G. F. Rodwell, late Science Master in Marlborough College." By FRANK RUTLEY, F.G.S., Lecturer on Mineralogy in the Royal School of Mines. Communicated by Professor T. G. BONNEY, D.Sc., F.R.S., &c. Received May 18, 1886.

[PLATES 3—5.]

In this paper an endeavour is made to show the nature of the changes which have resulted from the action of heat upon certain vitreous rocks. The changes which take place in such rocks through natural processes may sometimes be effected by heat alone, at others by heat in presence of moisture. Of these actions the latter is probably the more frequent, but, at the outset, it seems important to ascertain the action simply of dry heat before studying the more complicated conditions engendered by the presence of water and the pressure of superincumbent rock masses.